measurement processes are less than ideal or because information about the state is less than complete. In the quantum case, even given ideal measurements and a precise specification of the state, we obtain nonextremal values of probability.

Thus, in the state-space models we supply for determinist (classical) processes on the one hand and inherently probabilistic (quantum) processes on the other, the distinction between them appears neither as a radical divergence between accounts of the evolution of states, nor simply as a distinction between descriptive and dispositional accounts of states. It appears as a difference between the kinds of predictions a state makes available. Only in the determinist case are these predictions, as we say, *dispersion-free.*

But at this point we can scent a problem. Assume we have a quantum system Q and a measurement apparatus M. If the measurement process is to conform to quantum theory, we would expect the state of the coupled system Q + M to evolve according to Schrödinger's equation, that is, deterministically; nothing so far suggests that a complex system offers an exception to that equation. But if we associate different experimental results with different states of M (its "pointer readings"), and if the evolution of Q + M is deterministic, how is it that results have probabilities other than 1 or 0? I postpone discussion of this question to Chapter 9; for the present, a faint whiff of the problem of measurement can be left to hang in the air.

2.8 *Theories and Models*

Table 2.1 shows how states and observables are represented in quantum theory; in Section 2.7 we saw how the time-evolution of states is expressed in terms of the action of a family of unitary operators on the vector representing the state. Quantum mechanics, we may say, uses the *models* supplied by Hilbert spaces.

Implicit in this way of presenting quantum mechanics is a general account of scientific theories. A theory T displays a set of models within which the behavior of ideal "possible systems" (or "T-systems") can be represented. For a realist, at least, to accept T is to say that there exist actual systems which are T-systems. (For an antirealist but still model-theoretic view, see van Fraassen, 1980.) The actual solar system, for example, is (approximately) a *Newtonian system,* that is, a system representable within the mathematical models supplied by the theory of classical mechanics. A system S is a *quantum system* if the behavior of S is representable within a Hilbert-space model in the way I have outlined.

This model-theoretic account of a scientific theory is by no means original—it can even be called "the new orthodoxy" in the philosophy of science. (See Suppes, 1967; Giere, 1979; Suppe, 1977, pp. 221–230.) It stands in contrast to "the received view" (the phrase is Putnam's: Putnam, 1962), which takes an axiomatic approach to theories and emphasizes the role of theoretical laws (see Suppe, 1977, pp. 3–61). While I don't quite share Schopenhauer's view of the Euclidean method (it is, he said, as if a man were to cut off both legs in order to be able to walk on crutches; Blanché, 1962), I would reject any claim that an axiom system is the ideal, canonical form for the expression of a scientific theory. The point is this. For any axiom system there exists a class of models; Peano's axioms for arithmetic, for example, have as a model the set of natural numbers. And within science we are not interested in axioms for their own sake, but in the class of models they define. It does not matter how this class is specified, provided that the specification is precise. When we investigate a theory, demands typical of the axiomatic approach — like the requirement that the specification be expressed in a first-order language, or that the predicates of this language be divided into two classes, observational and theoretical — give undue prominence to linguistic matters and are extraneous to our concerns. Thus van Fraassen (1980, p. 44):

> The syntactic picture of a theory identifies it with a body of theorems, stated in one particular language chosen for the expression of that theory. This should be contrasted with the alternative of presenting a theory in the first instance by identifying a class of structures as its models. In this second, semantic, approach the language used to express the theory is neither basic nor unique; the same class of structures could well be described in radically different ways, each with its own limitations. The models occupy center stage.

But when we say that quantum theory uses the models supplied by Hilbert spaces, what sort of models are these? They are models in two apparently dissimilar senses. In the first place, they are models as that term is used in contemporary mathematics; in other words, they are mathematical structures of the kind described in Section 1.8, containing sets of elements on which certain operations and relations are defined. More surprisingly, they are also models in the way that a Tinkertoy construction can be a model of the Eiffel Tower. Just as a point on the model can represent a point on the tower, so, for example, an operator on a Hilbert space can represent a physical quantity.

The two senses are linked in the following way. When we recognize that the Tinkertoy model is a model of the Eiffel Tower, we not only see that points on the model represent points on the tower, but also that certain important relations are preserved in this representation; for example, we would expect the ratio of the overall height to the length of one side of the base to be the same for both the tower and the model. That is to say, we expect the tower and the model to be isomorphic. But isomorphic structures are just the subject matter of model theory in the first, mathematical, sense.

The outline of quantum theory given in this chapter uses the mathematical structure of Hilbert space (a model in the first sense) to provide a representation (a model in the second sense) of the behavior of systems. This behavior has itself been described in very abstract terms; there is a wide gap between the way a working physicist uses quantum theory and the account of the theory I have offered. Of such accounts, Cartwright (1983, pp. 135–136) says,

> One may know all of this and not know any quantum mechanics. In a good undergraduate text these . . . principles are covered in one short chapter. It is true that the Schrödinger equation tells how a quantum system evolves subject to the Hamiltonian; but to do quantum mechanics, one has to know how to pick the Hamiltonian. The principles that tell us how to do so are the real bridge principles of quantum mechanics.

Cartwright gives an instructive account of how an inventive physicist bridges the gap by using models of particular processes "to hook up phenomena with intellectual constructions" (p. 144). "To have a theory of the ruby laser, or of bonding in a benzene molecule," she says, "one must have models for those phenomena which tie them to descriptions in the mathematical theory" (p. 159). These models, however, have a very different function from the mathematical model in which we represent states and observables. They are essentially models in the second, Tinkertoy, sense, which represent actual entities, like a ruby laser, in terms of fictional elements ("two-level atoms" in this instance) whose behavior is amenable to theoretical treatment. These are just useful representations, *simulacra* of what they represent, and are contrasted with the underlying mathematical theory: "a model — a specially prepared, usually fictional description of the system under study — is employed whenever a mathematical theory is applied to reality . . . Without [models] there is just abstract mathematical structure, formulae with holes in them, bearing no relation to reality" (pp. 158–159). This view of the mathematical theory is at odds with my suggestion that the mathematical models supplied by Hilbert spaces are also representational. Such models are not simulacra, nor are they to be contrasted with the theory; in fact, to present the theory is just to exhibit this class of models. In what sense, then, are they more than "abstract mathematical structures"? What, we may ask, do they represent?

Well, to ask this question is precisely to seek an interpretation of quantum theory. When we construct models of the Eiffel Tower or of the ruby laser, we start from these objects and proceed to the task of model building. In the case of quantum theory, we have certain notions like "state" and "observable" which find a representation in the model. Antecedent to the theory, however, these are very insubstantial concepts. We rely on the theory's models to tell us how they are to be understood. The process of interpreting quantum theory is thus the reverse of that of building a model of a preexisting object. We judge our models of the Eiffel Tower and the ruby laser by how well they represent the objects modeled. When we try to interpret quantum theory we assume that the representation the theory offers is a good one and ask Feynman's forbidden question: what sort of world could it represent? In the most abstract, perhaps metaphysical sense, what must the world be like, if it is representable by the mathematical models that quantum theory employs?

The Bell theorem has implications extending beyond the topic of HV theories, and I discuss these implications in Chapter 8. The conclusion to be drawn from it in this section is that no local HV theory for quantum mechanics is possible.

To sum up, any HV theory that reproduces the quantum-mechanical statistics must be both contextual and nonlocal.

6.9 *Interpreting Quantum Theory: Statistical States and Value States*

It seems that quantum mechanics cannot, via an appeal to hidden variables, be reformulated as a theory whose underlying phase space is classical. Furthermore, a straightforwardly classical interpretation of quantum theory itself is ruled out. Where, then, are we to look for another? Come to that, armed with thimbles and care, what exactly are we seeking? To obtain a more precise idea of what is involved in *interpreting a theory*, let us return to a suggestion made in Section 2.8, that to interpret quantum mechanics is to see what kind of world is representable within the class of models the theory employs.

Recall that, on the semantic view of theories, a scientific theory provides a representation, or model, of a certain domain. Thus geometrical optics provides a geometrical representation of the transmission, reflection, and refraction of light, the Bohr theory of the atom a model of atomic structure. Sometimes these models have a physical representation, sometimes they are wholly abstract mathematical structures, but in both cases they supply representations of the phenomena, or, as in the case of the Bohr model, of the structures postulated as underlying the phenomena. The Hilbert spaces of quantum theory are, obviously, of the second, abstract kind.

We interpret the theory by recognizing, in the models the theory provides, elements of a particular conceptual scheme. For example, in the Hamilton-Jacobi theory of classical mechanics for a single particle, the element ω of the phase space is interpreted as an encapsulated summary of the primary qualities of the particle, and the mathematical expression $-\nabla H(\omega)$ $[=(-\partial H/\partial x) - (\partial H/\partial y) - (\partial H/\partial z)$, where H is the Hamiltonian function for the system] is interpreted as the *force* acting on the particle, such forces being the efficient causes responsible for the processes the theory describes.

Thus the theory is interpreted within a particular *categorial framework*. I borrow the phrase from Körner (1969, pp. 192–210); a categorial framework is a set of fundamental metaphysical assumptions about what sorts of entities and what sorts of processes lie within the theory's domain. The *loci classici* for the articulation of the categorial framework of classical mechanics are Kant's *Metaphysical Foundations of Natural Science* and his *Critique of Pure Reason.* This categorial framework was well established prior to the appearance of the Hamilton-Jacobi theory; correspondingly, the task of interpreting the theory was that of looking for familiar sorts of things. If the fit between the categorial framework and the models that the Hamilton-Jacobi theory provided had been less than perfect—if, for example, there had been nothing in the model to correspond to the concept of a primary quality (or objective property), or if what was identified as an efficient cause had allowed a multiplicity of effects (or of what were identified as effects)—then the Hamilton-Jacobi theory would not have been classical mechanics.*

However, in the case of quantum mechanics, a very different situation obtains. The theory uses the mathematical models provided by Hilbert spaces, but it's not clear what categorial elements we can hope to find represented within them, nor, when we find them, to what extent the quiddities of these representations will impel us to modify the categorial framework within which these elements are organized. To interpret the theory is to articulate the categorial framework whose elements have their images within it; we obtain an interpretation by the dialectical process of bringing to the theory a conceptual scheme, and then seeing how this conceptual scheme needs to be adjusted to fit it. Because there are several solutions to this problem, there can be competing interpretations of the same theory. (Compare Holdsworth and Hooker, 1983, who talk of one "quantum mechanics" but several "quantum theories.")

The concept of a *property* can serve to illustrate this rather abstract discussion. Does quantum mechanics allow us to say that a system "has properties"? Certainly we can find represented in Hilbert space values of physical quantities: the subspace L_a^A (equivalently the projector $\mathbf{P}_a^A$) represents the value a of the observable A. But if these subspaces are to be interpreted as properties, then, in addition to the now familiar state represented by a density operator (and called variously the *statistical state* [Kochen, 1978] or the *dynamical state* [van Fraassen, 1981b]), a *value-state* λ (alternatively, a *micro-state* [Hardegree, 1980]) must be attributed to the system. Regardless of whether the statistical state is thought of as applying to individual systems or to ensembles of systems, the value-state must be thought of as applying to individual systems. The value-state will be purely descriptive; whereas the statistical state assigns a probability to each pair (A,a) (regarded as an experimental question), the value-state will specify at any juncture

* "Classical mechanics" is here identified with a class (T_1,I_1), (T_2,I_2), . . . of theories and interpretations.

which of these pairs can be regarded as the system's properties. A value-state will thus map pairs (A, a) onto 1 or 0, depending on whether the system possesses the property in question or not, and so will resemble a classical state.

Two remarks need to be made about this value-state. In the first place, the attribution of properties it provides is over and above the work done by the theory *simpliciter*. We use it to yield an interpretation of the theory which accommodates the notion of the properties of a system, but another alternative is always open to us, that of finding a categorial framework in which the notion does not appear. Second, even if we hang on to properties, the concomitant value-states cannot be just like their classical counterparts. For Kochen and Specker's theorem tells us that, for most quantum systems, there can be no function λ mapping all pairs (A, a) onto 1 or 0 in accordance with PVP — in other words, so that for each A there is exactly one value a for which $\lambda(A, a) = 1$. Any workable account of a value-state must therefore be modified away from adherence to PVP. Different modifications will yield different interpretations of quantum theory.

A number of these interpretations can best be explicated using the vocabulary of "quantum logic"; partly for that reason the next chapter is devoted to that topic.